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Studies of the Physical Properties of the Moon and Planets

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PREFACE

This report is submitted in partial fulfillment of RAND Corporation's obligations under Contract N-33561 (NASw-6) with the Jet Propulsion Laboratory, California Institute of Technology. It collects some of the results of the second three-month phase of RAND studies of the physical properties of the Moon and planets.

During the next quarter research will continue to be pursued in a broad range of planetary studies.

The technical work is centered in the Department of Planetary Sciences. Many RAND consultants and staff members have contributed conceptually to the research. Chapters for this report were contributed by the following:

Consultants, Yale Mintz and Zdenek Sekera (University of California, Los Angeles),

Staff Members, M. H. Davis, D. Deirmendjian, J. W. Kern, T. W. Mullikin, G. F. Schilling, and A. G. Wilson.

The material was compiled and edited by M. H. Davis.

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I INTRODUCTION

As in the first quarter, a major fraction of the RAND research effort during the second quarter centered on studies pertinent to the scientific instrumentation to be carried on the first Venus space probe. In addition, research is progressing in more general studies of the physical characteristics and environments of the near planets.

Chapter II of this report gives the background of RAND participation in the Venus probe experiment and a summary of pertinent activities during the past quarter. The remainder of that chapter is devoted to a description of other research undertaken under the auspices of the JPL contract and to a discussion of the specific research results, which are documented in the form of brief technical notes or abstracts of longer papers in Chapters IV-XI of this report. A summary is also provided of visits to RAND under the auspices of the RAND--JPL contract by scientists from other organizations.

Chapter III summarizes briefly certain current studies underway within the RAND Corporation under other contracts which deal in some way with the space environment and the physical properties of the planets. Chapters IV-XI present abstracts and technical notes which document part of the research of the past quarter.

II REVIEW OF RESEARCH UNDER THE RAND--JPL CONTRACT DURING THE SECOND QUARTER

Early in the quarter, the NASA Subcommittee on Planetary and Interplanetary Sciences decided against the light scattering--polarization experiment proposed by D. Deirmendjian and Z. Sekera for Venus Space Probe P-37. A description of this experiment can be found in the first RAND Quarterly Progress Report, RM-2661-JPL, Sept. 30, 1960. However, the Subcommittee, following the recommendation of JPL, appointed D. Deirmendjian and Z. Sekera, along with C. Barth (JPL) and J. Chamberlain (Yerkes Observatory) as experimenters on the ultraviolet spectrometer to be flown on the Venus space probe. A preliminary evaluation of the scientific merits of such an instrument and what it might achieve had already been provided to JPL by RAND. (See Chap. VI by W. W. Kellogg in RM 2661.)

During the quarter a number of discussions of the ultraviolet spectrometer were held within the RAND Planetary Sciences Department, and with JPL representatives. December 5, at a meeting held at RAND, the instrument and the types of data it can be expected to provide were discussed along with the role of the various experimenters in the analysis of the data. Those attending were R. Davies, D. Margetts, C. Barth, and R. Newburn, all of JPL; J. Chamberlain of Yerkes, L. Wallace of the University of Chicago, M. H. Davis, D. Deirmendjian, and G. Schilling of RAND, and Z. Sekera of UCLA, RAND consultant.

One result of the meeting was agreement to align the entrance slit of the spectrometer so as to be parallel to the Venus terminator on the most probable trajectory. After the meeting, most of those attending visited the Space Technology Laboratories' installation in Canoga Park to see the instrumental development first hand.

M. H. Davis, D. Deirmendjian, and Z. Sekera visited JPL on December 21 for further discussions with R. Newburn, D. Margetts, and D. Spurlin. An outcome of these two meetings was a letter from Deirmendjian and Sekera to Newburn, the Assistant Project Scientist, outlining their position on the spectrometer design, and indicating the results they hope to achieve from the experiment. This letter has been rewritten as a technical note and appears in this report as Chapter IV.

Throughout this period, Deirmendjian and Sekera have been in close contact with Dr. Newburn and have responded to several requests for specific assistance; in particular, for estimates of light intensities to be expected from emission and scattering by the Venus atmosphere, and for an appraisal of the spectral region required for their analyses.

Besides the activities just described, which pertained directly to the ultraviolet spectrometer, research has been in progress on more general aspects of planetary sciences.

Y. Mintz, of the UCLA Meteorology Department and a consultant to the RAND Planetary Sciences Department, is conducting a broad program of research in planetary meteorology. He is at present preparing a

paper on theoretical models of the atmospheric circulation of Mars and Venus. An abstract of this paper, which is nearly complete, is included as Chapter V of this report. Professor Mintz has also been concerned with the interpretation of the Venus microwave radiometric data. His views on the Venus atmosphere, which are not easily reconciled with the very high surface temperature apparently implied by the radiometric measurements, were expressed in a letter to Dr. Douglas Jones of JPL. This letter, rewritten as a technical note, is included here as Chapter VI.

With an eye to space probes that will include atmospheric entry packages, G. Schilling makes the point in Chapter VII that from atmospheric data currently available, it appears that the engineering problems for Mars entry are considerably easier than those for Venus entry. The decelerations expected in Mars atmospheric entry are nearly an order of magnitude smaller, and the maximum heating loads, significantly less. These ideas were expressed in a letter to Dr. M. Eimer of JPL.

A very general epistemological study is under way by A. Wilson of the Planetary Sciences Department on how best to exploit the scientific means that are becoming available for the exploration of space. A brief discussion of a new way of looking at planetary experiments based on these ideas is presented in Chapter VIII.

An analysis was made by M. H. Davis and G. Schilling of the orbital behavior of the Echo I artificial earth satellite. On the basis of this analysis, an estimate was made of atmospheric densities in the earth's exosphere. A significant conclusion drawn is that the

concept of a "model atmosphere" is still useful for the exosphere. This work is reported in Chapter IX, which will become a technical paper for the open literature, possibly after some amplification. Research such as this, since it reveals new results with regard to the earth's exosphere, has important implications with regard to the space environment near Mars and Venus.

In the interpretation of the data from a planetary probe, one problem will be to visualize the trajectory and the angular relationships between the probe, the target planet, and the sun. J. Kern has devised a method for presenting trajectory information in a form which is at once simple, quickly interpreted and immediately useful. Chapter X outlines this method, which derives from the stereographic projection methods commonly used in geology. The method is illustrated by taking as a specific example, a preliminary trajectory provided RAND by JPL for the Venus P-37 probe.

Some ideas on the determination of the trajectory of a space probe from on-board observation made in the vicinity of the target planet are presented in Chapter XI by T. Mullikin. The method discussed does not appear applicable immediately but may serve as a basis for developing other interesting attacks on the problem.

Visitors to RAND under JPL contract support. During the last week in September, 1960, Dr. T. Gehrels of the Astronomy Department of the University of Indiana visited RAND at the invitation of D. Deirmendjian. He delivered a lecture on his measurements of the polarization of the light from Venus and presented ideas on possible

observations of planets from balloons. Informal discussions were held with members of the Planetary Sciences Department, particularly with D. Deirmendjian, on scattering by planetary atmospheres.

November 9, Dr. A. Dollfus of the Paris Observatory visited RAND for informal discussions. He also delivered a lecture on his observations of the polarization of light from Mars.

Dr. D. Jones of JPL visited RAND on December 5, 1960 to discuss his interpretation of the microwave radiometric observations of Venus with P. Goldberg, Y. Mintz and other RAND staff members and consultants.

Dr. Fred Whipple, the director of the Smithsonian Astrophysical Observatory visited RAND for several days in December for discussions with members of the Planetary Sciences Department. He presented two lectures, one on the orbiting telescope for ultraviolet survey, the other a review of his recent work on meteoric dust. Dr. Whipple is a consultant to the RAND Planetary Sciences Department.

III SOME PROJECT RAND ACTIVITIES RELATED TO THE RAND--JPL CONTRACT

Since there are many studies being pursued at RAND under Project RAND and other projects that deal in some way with the space environment and the planets, the discussion here is included merely to provide a brief description of certain of these activities that may be of particular interest to JPL.

A research memorandum, RM-2567, "Summary of Orbital and Physical Data for the Planet Mars" by D. Kirby was prepared last summer under Project RAND. Some further work is in progress whose object is to outline the bibliographic sources of planetary data needed for engineering studies and generally to relate what is known or suspected by the astronomer with what is needed by the engineer.

H. Schechter has made an extensive study of ways to compute impulsive 3-dimensional interplanetary trajectories simply and rapidly. The work has been reported in a RAND paper, P-2157, "Determination of Interplanetary Transfer Orbits for Specified Date of Departure," December 1960. In the next few months, he also expects to be investigating low-thrust trajectories.

B. Boehm and R. T. Gabler have developed a general atmospheric-entry program for the IBM 7090, which has found extensive use within RAND. This program might be useful in the investigation of the trajectories of entry capsules.

Among the activities of the Aerodynamics Group of the Aero-Astronautics Department, there has been some research into the thermodynamics of possible Venus atmospheres. Earlier work is summarized in RM-2292, "Thermodynamic Properties of Carbon Dioxide to 24,000° K with Possible Application to the Atmosphere of Venus," by J. L. Raymond, November, 1958. Recently, L. D. Kaplan of MIT, a RAND consultant, has brought together in a novel way some recent data dealing with the Venus atmosphere. His results are to be published in a forthcoming paper. Dr. Kaplan is also monitoring current literature for RAND, seeking new data and methods of analysis to derive clues about the nature of the Venus atmosphere. During the summer of 1960, another consultant, Mr. W. Strahle, began a calculation of the thermodynamic properties of a possible Venus atmosphere composed of 85 per cent CO_2 and 15 per cent N_2 by volume. In addition, the normal shock calculations were set up for selected altitudes and speeds and are now being calculated on the IBM 7090 computer.

D. Deirmendjian is continuing work on the theory of Mie scattering by particles with finite conductivity. A paper and a RAND research memorandum now in preparation deal with this problem.

A study of the capture of meteoric material by a planet is being conducted by S. Dole, considering the fundamental 3-body nature of the problem. He is also carrying on a long term study in what might be called "general planetology". The object is to systematize the study of planets so that a particular planet is not regarded as an isolated entity, but rather as a member of the general class of planetary objects with such characteristics as a particular rotation rate, angle of

inclination of the axis of rotation, orbital eccentricity, and mean distance from a star of a certain type. A general study such as this can be expected to yield important concepts relating to planetary exploration.

IV--REMARKS ON THE ULTRAVIOLET SPECTROMETER FOR THE MARINER A

D. Deirmendjian and Z. Sekera

The following observations represent the authors' current thinking on the ultraviolet spectrometer to be carried on space probe *Mariner A*.

1. Slit Orientation

It is planned that the slit will be oriented parallel to the terminator. All concerned agree that this is desirable.

2. Scanning the Planet

Indications are that the direction of spot scanning chosen will be at 15 degrees inclination to the chord through the cusps. The authors agree that this is a good mode, since the detection and mapping of aurorae is one of the main objectives of the experiment.

Let us assume that the problem of stray light has been solved and that the primary aperture of the system receives light from a more-or-less well defined area of the planet. Suppose further that the number of spot measurements during a scan is constant and predetermined but not tied to position with respect to the terminator line; in other words, the area observed may be entirely in the day or night hemisphere, it may straddle the terminator, or it may be mostly the night side with only a small percentage of twilight zone. The latter case may very well occur with the present scanning mode, especially

at planetary phase angles exceeding 90°. Then the light received by the spectrometer will be dominated by characteristics of the twilight zone. The resulting spectrum, covering the whole region to 6500Å, will be of utmost importance, both for the study of day glow and possible sunlit auroral features and for the detection of oxygen and ozone in absorption. Such a spectrum would be easily identified with the twilight zone, since it should be quite different in intensity and shape from the pure day- or night-side spectra.

3. Spectral Range

It has been suggested that the experiment be limited to wavelengths shorter than 4000Å. If this becomes necessary, ozone can still be detected from day-side spectra in the region 2500--3100Å with a 10-angstrom resolution. This suggestion is based on a recent detailed analysis of the terrestrial case (Z. Sekera and J.V. Dave, "Diffuse Reflection of Solar Ultraviolet Radiation in the Presence of Ozone," Sc. Rep. No. 4, UCLA, 1960).

We believe that the contractor's hands should not be tied by insistence at this early stage that the visible region of the spectrum be omitted. For the day side, the problem of sensitivity range could be solved by using a neutral filter or by reducing the entrance slit whenever the light level exceeds a certain preset limit. For an adequate analysis of the visible light from the sunlit part, it is not essential that all wavelengths in the 4000--6500Å region be recorded and transmitted. It may be possible to preselect and transmit only a set of wavelengths around certain regions; for example, around 4000, 4500, 5000, 5500, 6000, and 6500Å; i.e., six values from each spectral scan. This procedure may reveal the weak but

important Chappuis band system of ozone, especially at large phase angles. Comparison of data of this type with data from the Hartley-Huggins regions should improve our estimate of the amount of ozone in the atmosphere of Venus.

We understand that the grating is to be blazed in the ultraviolet. We must point out that such a blaze would decrease the intensity of the night airglow and auroral spectra in the visible region. Limiting the useful range to under 4000Å eliminates the important green and red oxygen emissions, and if the twilight zone is in fact observed, any sodium resonance scattering would be missed.

4. Calibration

Best calibration against laboratory standards should be sought. Other preflight calibrations are desirable, such as testing the spectrograph at the site of current terrestrial airglow and aurora work (University of Alaska, National Bureau of Standards, Colorado) or even on the ultraviolet sky light for Umkehr ozone measurements. For calibrations in flight, an attempt to include at least one solar spectrum is highly desirable.

5. Stray Light

Reduction or elimination of stray light is of very great importance for the night-side spectra, but it is a difficult problem in view of the large aperture of the instrument and the limitations in the size of the package.

A more acute problem, however, is that of stray visible light

from the day side. Two methods of eliminating or reducing the stray light come to mind. One is an external filter with low transmission in the visible band, but it seems that such filters are unavailable, and in general, any filter is undesirable because it cuts down the energy in all bands.

The other method is to make an ultraviolet-sensitive photomultiplier the main sensor; in this way the visible stray light is reduced to a minimum even if it enters the optical system. If the photomultiplier is used, an auxiliary detector with maximum sensitivity in the visible should also be carried, to which the visible part of the spectrum can be deflected by an optical flat or other means whenever the day side is being scanned.

For the sake of avoiding confusion, it is here suggested that the term "scattered light" not be used in reference to this problem. The terms "stray light" and "internal reflections" appear less ambiguous, and the term "scattered light" should be used only when referring to sunlight scattered by the planet's atmosphere, before entering the optical system.

6. Experimenter's Role

It is not enough that the data to be analyzed be given to experimenters competent in the field. For its effective interpretation, an *a priori* division of the data, such as peaks versus valleys, or ultraviolet versus visible, should be avoided. The observed spectra will be a result of emission, absorption, *and* scattering. From terrestrial experience, we expect one or two of these mechanisms to

predominate at certain wavelengths and positions on the disk. The incoming data will be the first of their kind, and only their inspection can tell us what mechanisms are involved individually or in combination. A corollary to this important provision is the necessity for close cooperation among the experimenters during the period of data analysis.

The authors hope to get the following information from the Venus spectrometer experiment if performed as recommended above, including a partial scan of the visible part of the spectrum taken at various planetary phase angles.

a. Detection, and possible determination of the amount, of ozone in the Venus atmosphere; also, if twilight data are obtained, knowledge of the relative position of the ozone layer.

b. Detection of molecular oxygen in the uppermost layers.

c. Detection of other gaseous constituents with strong absorptions in the ultraviolet.

d. If ozone is detected, the relative depth of the underlying gaseous atmospheres above the clouds or surface.

e. From the character of the ultraviolet and visible continuum, a determination of whether or not the atmosphere above the clouds or surface is gaseous, or whether there are high cloud or dust layers as proposed by Edson and others.

f. A lucky scan may indicate specular reflection from the underlying surface.

If the spectrum is limited to wavelengths shorter than 4000Å, items e, f, and possibly d, will be unattainable.

It is clearly understood that the spectrometry experiment does not have the same mission or design as the photometry and polarization experiment proposed by one of us several months ago. We believe that the present experiment is well designed for its primary mission. The object of this chapter is to show that considerably deeper layers of the atmosphere of Venus can be probed with little additional effort.

V ON THE GENERAL CIRCULATION OF THE PLANETARY ATMOSPHERES**

Yale Mintz

The two fundamental forms of general circulation of a planetary atmosphere, the symmetric Hadley regime and the Rossby wave regime, are described. It is shown that both circulation regimes transport heat from the low-latitude heat source to high-latitude cold source, and that both regimes produce the kinetic energy necessary to maintain the circulation against frictional dissipation.

Using a two-layer model of the atmosphere, it is shown that thermal equilibrium will be established in the symmetric regime when

$$\left(-\frac{\partial T_2}{\partial \phi}\right) = \left(-\frac{\partial T_2}{\partial \phi}\right)_{H=\Delta Q} = \left(\frac{\sqrt{2}\,\Omega^2\, m}{2^K\, \pi\, g\, R^*\, \mu\, s}\right)\Delta Q,$$

$$K \equiv R^*/mc_p\ ,\ s \equiv (1 - \gamma/\gamma_d)$$

where

T_2 = the temperature at the middle level of the atmosphere

ϕ = latitude

Ω = rotation rate of the planet

g = acceleration of gravity at the surface of the planet

R^* = universal gas constant

c_p = specific heat at constant pressure

**Summary of a paper in preparation.

m = mean molecular weight of the atmospheric gas

μ = vertical eddy viscosity

ΔQ = net heating of the atmosphere between the equator and the central latitude

γ = vertical lapse rate

γ_d = adiabatic lapse rate

But it is also shown that the requirement for dynamic stability of the symmetric regime is

$$\left(-\frac{\partial T_2}{\partial \phi}\right) < \left(-\frac{\partial T_2}{\partial \phi}\right)_{crit} = K T_2 S .$$

It follows that, if

$$\left(-\frac{\partial T_2}{\partial \phi}\right) = \left(-\frac{\partial T_2}{\partial \phi}\right)_{H=\Delta Q} < \left(-\frac{\partial T_2}{\partial \phi}\right)_{crit} ,$$

the symmetric circulation will be in thermal equilibrium and dynamically stable.

But if
$$\left(-\frac{\partial T_2}{\partial \phi}\right)_{H=\Delta Q} > \left(-\frac{\partial T_2}{\partial \phi}\right)_{crit}$$

and
$$n_{crit} \geqslant 1 ,$$

then, as shown in the paper, the symmetric regime will be unstable, small disturbances will grow and the circulation will change into the wave regime, with the number of waves around the planet given by

$$n_{crit} = a\,\Omega\,\frac{m}{R^*}\left(\frac{\sqrt{2}\,C_p}{T_2\,S}\right)^{\frac{1}{2}} ,$$

where a is the radius of the planet.

In the wave regime thermal equilibrium is quickly reached,

when
$$\Delta Q = \mathcal{R} = \frac{\Omega p_s}{2\pi g K} n \left(\sin\alpha\right) V_2^2 \quad ,$$

where

$\mathcal{R}$ = the net poleward heat transport by the waves, at the central latitude

p_s = the surface pressure

n = the wave number

α = the phase difference between the isotherms and the streamlines of the waves

V_2 = the amplitude of the waves at the middle level of the atmosphere

But the wave regime, unlike the symmetric regime, is not a steady-state circulation. It is a stable regime of circulation only in the sense that it is "permanently convective," with new waves growing when the old ones die out. Thermal equilibrium is maintained only in the long-term average.

In the paper, the theory is tentatively applied to the atmospheres of Mars and Venus as well as to the Earth (using for Mars and Venus what the author believes are reasonable guesses for the critical parameters S, μ and Ω) with the following results:

On Earth, according to the theory, in both winter and summer the stable form of the general circulation is the wave regime with wave number 5 or 6 -- which is in good agreement with observations.

On Mars, in the winter hemisphere the stable circulation is the wave regime with wave number 3. But in the summer hemisphere of Mars

the symmetric regime is the stable one.

On Venus, assuming $\Omega < \frac{1}{5}\Omega_E$, the critical wave number is less than one, and therefore, regardless of the relative magnitudes of $\left(-\frac{\partial T_2}{\partial \phi}\right)_{H=\Delta Q}$ and $\left(-\frac{\partial T_2}{\partial \phi}\right)_{crit}$, the symmetric circulation is the stable regime throughout the year.

VI A NOTE ON THE INTERPRETATION OF RADIOMETRIC MEASUREMENTS OF VENUS

Y. Mintz

The evidence of the centimeter radiation from Venus indicating that the surface temperature is very high seems difficult to reconcile with the analysis of the temperature and circulation of the Venus atmosphere presented in RAND Paper P-2003 (Temperature and Circulation of the Venus Atmosphere, Y. Mintz, May 27, 1960). For this reason, the current theoretical work of Dr. Douglas Jones of JPL on possible radio emission by the Venus ionosphere is particularly important. In this regard, attention should be drawn to an interesting feature of the 8-mm radiation as given in the paper by A.D. Kuz'min and A.E. Salomonovich, "Radio Emission from Venus in the 8-mm Bandwidth", Soviet Astronomy, 4, 279 (1960). In this paper, it is shown that, when the phase of Venus changes from inferior conjunction to one in which about a third of the disk is illuminated by the sun, the brightness temperature averaged over the whole disk increases by about 100°K. This indicates that the local brightness temperature of the sunlit side is a few hundred degrees higher than that of the dark side. It is very difficult to believe that the true surface temperature of the planet can vary by this amount - especially when the 10-micron radiometric temperature varies hardly at all from the sunlit to the dark side.

A more plausible interpretation is that the ionospheric temperature increases with elevation (let us say, by something of the order of 5 to 10° per kilometer) and that the electron density is greater on the sunlit side than on the dark side to the extent that the effective level of the microwave emission is of the order of 50 to 100 km higher in elevation on the sunlit side than on the dark side - and, hence, the greater observed brightness on the sunlit side. This would imply, also, a change of the longer microwave radiations with the phase of the planet.

But, alternatively, one might regard the day-to-night difference in intensity as due to the 8-mm wavelength being exactly in the transition zone, so that during the night the 8-mm radiation comes mainly from the visible cloud level of the planet; but during the day, with the increase of ion density, the critical wavelength shifts to a shorter value and the 8-mm radiation then comes mainly from the high temperature ionosphere. In this case the longer wavelengths would not show the change with phase of the planet.

VII A NOTE ON ATMOSPHERIC ENTRY: MARS VERSUS VENUS

G. Schilling

Considering the landing on a planet as one of the long-range objectives of our space program, it would appear that, for economy of engineering effort, such efforts should be concentrated on Mars rather than Venus. In comparison to Earth or Venus, entry into the Martian atmosphere apparently requires considerably lower decelerations, because of both the lesser planetary mass and the more gradual variation of density with altitude. Quoting from recent work by Carl Gazley Jr. (Atmospheric Entry, RAND Paper P-2052, 15 July 1960), deceleration based on currently accepted models for planetary atmospheres, is generally one order of magnitude more favorable for Mars than for Venus. The numerical values below are indicative (values for Venus in parentheses).

Entry Velocity	Entry Direction (Degrees From Horizontal)	Maximum Deceleration (g)	Maximum Heating (Degrees K)*
Escape	90	18.3 (326)	1800 (3500)
	5	1.6 (28.6)	1300 (2600)
Orbital	90	9.2 (163)	1400 (2700)
	5	0.8 (14.3)	1100 (2100)
Decay from satellite orbit		1.4 (8.9)	1100 (1900)

*Equivalent radiation equilibrium temperature for m/C_DA = 50 kg/sq.m.

It should be remembered that new information on the constituents of the atmospheres of Mars and Venus could, of course, modify these estimates.

In future planning, the following arguments will be relevant:

- Development and initial employment of an instrumented sounding capsule (drop sonde) for a 1964 Venus probe, in all likelihood, would be a difficult engineering development in spite of its high scientific desirability.

- Development and initial employment of a landing capsule for a 1964 Mars probe, on the other hand, should have a high probability of success within the given time schedule.

- In addition, for a 1964 Mars drop sonde, it may prove possible to consider geophysical landing experiments such as are under development for the lunar probes (seismometer, gamma-ray spectrometer, etc.) in addition to atmospheric experiments.

VIII TOWARD A STRATEGY OF SCIENTIFIC SPACE EXPLORATION

A. Wilson

The scientific exploration of space presents a plethora of problems involving decisions such as: what are the central and most significant questions which the scientific program should seek to answer, what observations will best provide the answers, in what order should observations be made, with what instruments, from what locations, etc., etc. The complex and interlocking nature of these questions certainly requires that some sort of a strategy of scientific exploration be derived that can provide criteria for making decisions leading to the best program for acquiring significant knowledge.

Although this problem is orders of magnitude more complex than the usual dynamic decision-making configurations encountered in industry, it should be possible through a systems approach to identify the competing requirements and isolate some of the specific problems to be solved.

A system can be visualized whose structure consists of material components such as instrument carriers; sensors; together with data storage, filtering, and transmission devices; and also abstract components such as scientific hypotheses, experimental techniques, and reduction processes. This is a system whose primary function is to acquire - not just data - but knowledge. It may be appropriately termed an epistemological system.

A problem arising in the comparison of various epistemological systems is to devise a measure of their relative capabilities which will constitute a figure of merit for specific assignments. One such useful

measure is provided by the concept of an <u>information space</u>.

As an introduction to the idea of an instrument-system information space, the process of direct photography provides a good example.

The principal parameters of a photographic system may be regarded as dimensions. These are the two linear or angular dimensions of the region photographed, the brightness dimension (which is recorded as photographic density), the spectral dimension, and the temporal dimension. In each of these dimensions there are bounds which define the <u>range</u> and a threshold which sets the <u>resolving power</u>. Table 1 defines these quantities for each parameter.

Although there is not complete symmetry among the parameters when considered in this way, we may none the less construct a five-dimensional "Instrument Information Space" whose extension is determined by the ranges, and which can be divided into information cells whose sizes are determined by the several resolving powers.

Direct photographic observations of planets are constrained by the following four factors:

1) the optical and photographic parameters such as telescopic aperture, emulsion sensitivity, grain, contrast, etc.;

2) relative motions of the instrument and field being photographed;

3) the location of the instrument with respect to the planet being explored;

4) considerations of technical and economic feasibility.

Item 1 is fully discussed by de Vaucouleurs.* Item 2 refers to the dynamic stability of the telescope-camera system, periods of natural

*"Planetary Astronomy from Satellite-Substitute Vehicles," Chap. II, AFMDC-TR-60-6.

oscillations, guidance, and to relative motions caused by planetary rotations, movement of the instrument carrier, etc. Item 3 involves the limiting effects of seeing, sky brightness, and spectral transmission properties of the earth's atmosphere, also the effects of distance to the planet on the linear field of view and linear resolving power. Item 4 involves the sizes of instruments which may be practically carried in balloons, placed in orbit about the earth or carried in fly-by probes and planetary landing capsules. It also involves the cost constraints of each system and the extent of program economically feasible with each system.

Table 2 gives illustrative comparative instrument information spaces for Mars for a telescope of 60-inch aperture on the earth's surface, a 20-inch telescope in a balloon at 100,000 feet and a 5-inch telescope in a fly-by probe 10^6 km and 4×10^4 km from the planet.

Table 1

DIMENSIONS FOR PHOTOGRAPHIC SYSTEMS

Dimension	Resolving Power	Range	Effective Range
Areal (2 dimensions)	Angular resolving power set by telescopic and photographic parameters together with seeing and instrument stability limitations.	Determined by angular field of view.	Field of view, modified by aberrations of the optical system.
Brightness	The contrast, γ	Determined by signal-noise ratio and emulsion saturation.	Determined by a set of exposure times.
Spectral	The filter emulsion band pass.	Determined by filter-emulsion combinations and optical and atmospheric transmission limits.	Sums of bands at which exposures are taken.
Temporal	Exposure time and/or frequency of exposures.	Span of observations.	Determined by number of exposures, divided by frequency of exposures.

Table 2

COMPARATIVE SYSTEMS INFORMATION SPACES FOR PHOTOGRAPHY OF MARS

	For Mars at Most Favorable Opposition: Distance 56×10^6 km		For Probe with 5" Telescope	
	(A) 60" Telescope at Earth's Surface	(B) 20" Telescope Balloon Mounted	(C) Probe at 10^6 km from Mars	(D) Probe at 40,000 km from Mars
Angular Field	Entire planet 25" arc.	Entire planet 25" arc.	20' arc entire planet	10° entire planet
Angular Resolving Power	Theoretical optical, 0".09; seeing limitation 0".5 to 2"	Theoretical optical, 0".27; guidance limit 0".01	Theoretical optical, 1".1	Theoretical optical 1".1
Linear Resolving Power	Theoretical optical, 25 km; seeing 140 to 560 km.	Theoretical optical, 75 km; guidance 3 km.	5 km	0.2 km
Spectral Range	0.32μ to 0.9μ	0.29μ and longer	Limited only by Martian atmosphere	
Exposure Time	Signal: Noise limit to seeing and emulsion saturation limits. Rotation of Mars limit. 0.01 sec to 2 min.	Signal: Noise limit to emulsion saturation rotation limit. 0.01 sec to 2 min.	Limited by the velocity of the probe relative to the surface of Mars	
Frequency of Exposures	Any frequency up to reciprocal of exposure time.	Same as surface except for uneconomical frequency interval of one exposure every hour to 100 hours.	Up to reciprocal of exposure time.	
Number of Exposures	Limited by economic and data processing factors.	Limited by economic factors governing number of flights.	Limited by data storage and transmission capability and by frequency of exposures and allowable span of observations.	
Span of Observations	Limited by planetary configurations and sky brightness. Possible up to 45° from sun.	Daytime observations possible up to 5° from sun.	Determined by orbital parameters of probe	

IX DENSITY OF THE VERY HIGH ATMOSPHERE

M. H. Davis and G. F. Schilling

Knowledge of the high atmospheres of the nearby planets is so scanty that it is difficult to know where to begin in designing experiments to determine their nature. One starting point for such investigations is the exosphere of the earth. In this chapter, certain conclusions about the exosphere are derived from recent satellite data, and the suggestion is made that similar conditions may exist on the nearby planets.

From orbital data provided at approximately weekly intervals by the Smithsonian Astrophysical Observatory for Satellite 1960 IOTA ONE (Echo), we have inferred preliminary mean density values of the high atmosphere between 1000 km and 1500 km above the international geoid. The technique used was essentially that described earlier(1) and made use of Stern's approximation formula(2) programmed for the RAND Johnniac computer. However, because of the unique orbital behavior of the Echo Satellite, logarithmic density gradients or scale heights did not have to be assumed but could be derived directly from the data.

Table 1 shows the computed mean atmospheric densities. Tracking inaccuracies and possible variations of the mass-area ratio of the satellite produce an uncertainty of about a factor of 2 in the tabulated results. Radiation pressure on the satellite does not

affect the total energy of the satellite orbit and does not change the theory used for our density calculations. However, the resulting changes in perigee altitude permit the inference of densities at various heights as a self-consistent set of data from one single satellite.

It should be noted that the values given in Table 1 are mean values averaged individually over successive intervals of from six to sixteen days. During the total time interval, the position of perigee varied between latitudes 47° north and south, and passed in and out of the earth's shadow. In view of the preliminary nature of the data, however, it was not felt that an evaluation as to possible diurnal, latitudinal, or seasonal variations of the high atmosphere was yet warranted.

Fluctuations due to sporadic solar activity could be expected to show distinctly in these mean values only if the resultant effects on the earth's high atmosphere produce large and persistent changes in temperature and density. Major geomagnetic disturbances, as evidenced by the planetary magnetic three-hour-range indices, occurred around September 5, October 6, and November 13. If the density data are plotted against time, there are indeed some indications that the mean density values on reference dates following these magnetic storms are somewhat raised over undisturbed conditions. However, no definite conclusions regarding the influence of solar activity on exospheric density could be drawn from the data available to us.

The variation of the mean atmospheric density with altitude is shown in Figure 1. The mean gradient corresponds to an average

density scale height of about 273 km between 1000 and 1500 km above the earth's surface. For comparison purposes, the figure also contains portions of the RAND Model I Atmosphere for latitudes 0° and 45°, [3] and the Smithsonian Interim Atmosphere Number 2.[4] Examination shows a general consistency of the data over the almost four-month period with surprisingly little scatter.

On the basis of rocket and satellite data obtained at much lower altitudes, Jastrow and Bryant [5] predicted that the combination of diurnal fluctuations and solar effects would easily amount to density variations by two orders of magnitude at 1500 km altitude. If such exceedingly large fluctuations actually took place due to any cause, they should have shown in the mean values given here, unless they were of very short duration. A least-square curve of constant-density scale height was fitted to the data of Fig. 1. The percentages by which the data in Table 1 deviated from this curve at given altitudes on the dates cited are listed in Table 2. As is evident, these variations are of the order of 35 per cent rather than orders of magnitude.

It appears to us that at least in regard to the variation of density with height the physical state of the very high atmosphere is rather close to an equilibrium condition. Solar effects may produce noticeable deviations in temperature and density but with a relatively fast return to the mean condition. In consequence, standard model atmospheres can be used when an approximation of the "climate" of the very high atmosphere is desired. The most striking evidence for this

conclusion is implicitly contained in Figure 1, recalling that the early RAND Atmosphere Models were derived by Grimminger (3) some 12 years ago and that the Smithsonian Interim Atmosphere was a mathematically rigid extrapolation of just a few preliminary data from the first two Soviet satellites. To progress from the designation of climatic conditions to depicting and understanding the "weather" of the very high atmosphere is, of course, dependent on the accumulation and careful analysis of data obtained at frequent intervals over a long period of time.

Since we have found such an apparent consistency in the exosphere of the earth, it might be reasonable to speculate that similar conditions may exist in the high atmospheres of other planets.

Acknowledgment. We are indebted to Dr. Hilde K. Kallmann for pointing out the need for mean density values of the very high atmosphere to assist the COSPAR Working Group on International Standard Atmospheres.

Table 1

Mean Values of Atmospheric Density

Reference Date (1960 Datum)	Mean Perigee Altitude (km-MSL)	Mean Sub-Perigee Geocentric Lat. (Degrees)	Mean Density (g/cm^3)
Aug. 30.5	1,462.0	37°N	2.82×10^{-18}
Sep. 6.5	1,462.1	45°N	3.33×10^{-18}
Sep. 13.5	1,386.7	45°N	5.52×10^{-18}
Sep. 20.5	1,347.0	38°N	5.53×10^{-18}
Sep. 27.5	1,309.4	26°N	5.10×10^{-18}
Oct. 4.0	1,278.8	12°N	4.14×10^{-18}
Oct. 19.5	1,222.7	21°S	7.33×10^{-18}
Nov. 6.5	1,153.2	43°S	9.99×10^{-18}
Nov. 15.0	1,112.1	33°S	13.68×10^{-18}
Nov. 21.5	1,080.1	18°S	11.14×10^{-18}

Table 2

Normalized Density Variations

Time (Days after 1960 Aug. 30.5)	Normalized Density Deviation (Departure from Altitude Mean in Per Cent)
0	- 8 per cent
7	- 6 per cent
14	+ 36 per cent
21	+ 17 per cent
28	- 6 per cent
34.5	- 31 per cent
50	- 1 per cent
68	+ 4 per cent
76.5	+ 23 per cent
83	- 11 per cent

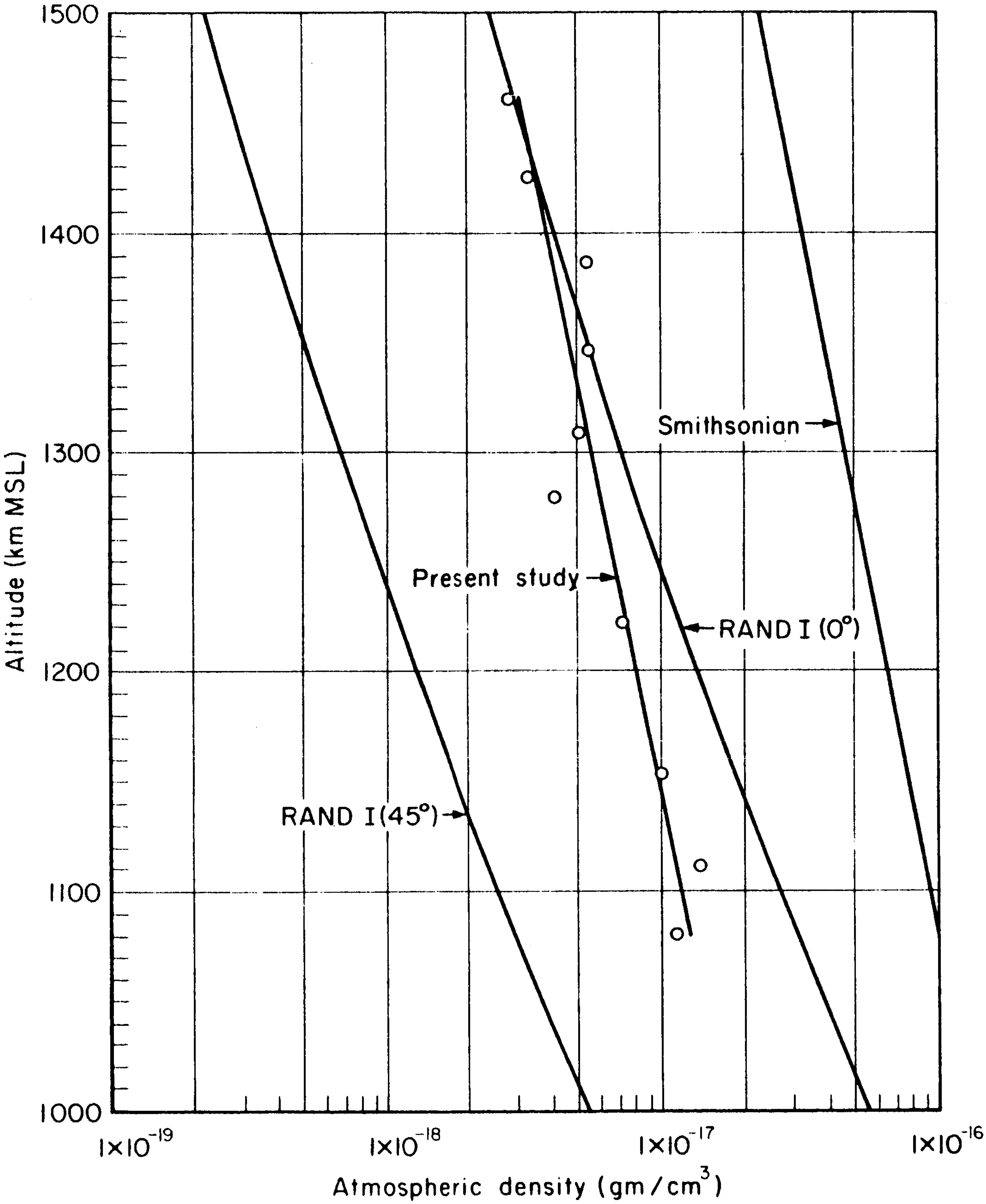

Fig. 1 - Variation of mean atmospheric density with altitude above earth's surface. Portions of model atmospheres are included for comparison.

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X REPRESENTATION OF ANGULAR RELATIONS IN A PROBE-CENTERED COORDINATE SYSTEM FOR NASA SPACE PROBE P-37 (VENUS FLY-BY)

J. Kern

To facilitate evaluation of interplanetary space-probe experiments, a form of representation is required that depicts the angular relations of the sun, the probe, and the planet in a convenient and understandable form. In an earlier work on this subject (Schilling and Deirmendjian, 1960)(*), a probe-centered coordinate system was selected. Such a coordinate system permits us to depict the appearance of the planet as seen by an observer on the probe at a given time. It is also possible to represent in such a coordinate system the relative angular positions of the Sun, Earth, and the target planet at any time. In this chapter, a form of representation for a probe-centered coordinate system is described that combines both of the features mentioned above, and hence has the property of depicting eclipse and telemetry cut-off points for trajectories that permit such events.

If a sphere of unit radius centered on the probe is imagined to surround the probe, the points where the surface of the sphere is pierced by lines from the probe to the target, to the earth, and to the sun, display the same angular relations with respect to the center of the sphere as the original lines. Planes in space through the probe center intersect the sphere in great circles. At any given time, an

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*Quarterly Technical Progress Report (I), Studies of the Physical Properties of the Moon and Planets, RM-2661-JPL, Sept. 30, 1960, pp. 54-60.

adequate description of the probe trajectory will permit construction of the angular relations noted above with respect to the probe. It is desirable, however, to have a planar representation of such a spherical surface which displays the angular relations and makes interpretation of such relations as simple as possible.

A form of generalized projection which has been used successfully for this type of problem is the stereographic projection.(*) The plane of projection is an equatorial plane through the center of the sphere. The points at which a line normal to a plane through the center of the sphere intersects the sphere are defined as the "poles" of the plane. The stereographic projection of any point on the sphere is the intersection of a line drawn through the point from a given pole of the sphere with a selected equatorial projection plane. In this manner any point on the hemisphere opposite the pole of projection can be represented on the projection plane by a point inside a unit circle with diameter corresponding to that of the sphere. In the standard stereographic projection, points on the hemisphere on the same side of the equatorial plane as the pole of projection project outside of the unit circle on the equatorial plane. An adaption of this projection technique is to change the pole of projection for this hemisphere to the opposite pole on the unit sphere. If this is done, both hemispheres can be mapped inside the unit circle on the equatorial plane, but this makes a convention necessary for identifying points on the projection with the hemisphere of origin.

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*See, for example, Higgs, D. V. and Tunell, G., Angular Relations of Lines and Planes, W. C. Brown, New York, 1959.

An important property of the stereographic projection is that all circles on the sphere project as circles or as straight lines on the equatorial plane. Circles on the sphere are of two kinds: great circles, corresponding to planes passing through the center of the sphere, and small circles, corresponding to planes which do not pass through the center of the sphere. All great circles through the poles of projection appear as diameters of the equatorial unit circle. All other great circles project as circles passing through the extremities of diameters of the equatorial unit circle. Another important property of the stereographic projection is that angles between great circles are projected in their true values. It is possible to utilize these two properties to develop construction techniques for solution of spherical triangles, and for the determination of angular relations between great circles. Plotting and measuring angles on a stereographic projection, and hence solution of problems involving great circles, is greatly simplified when a stereographic net is employed. An example of such a net is shown in Figure 1 due to G. Wulff. Called the Vulf net, the example shown illustrates great and small circles for every alternate degree, and can be visualized as the stereographic projection on a meridional plane of lines of geographic longitude and latitude.

Graphical solutions of problems dealing with angular relations between lines and planes in space may be obtained by plotting the lines and planes with the aid of such a stereographic net and reading the angles directly from the net. Graphical solutions with the aid of a carefully constructed stereographic net will be within the allowable limits of error for many problems. This is almost certain to be the

case for interpretation of many of the experiments from an interplanetary probe, since errors involved in solution of spherical triangles can be reduced to less than $\pm$ 1/2 degree on a net of 20-centimeter diameter. Nets are available with diameters exceeding 50 centimeters.

The stereographic projection can be conveniently used as a form of representation for the environment of an interplanetary probe. In particular, consider the appearance of the target planet as seen from the probe. In this case a reference axis may be taken along the direction from the probe to the planet. Then the poles of projection for a unit sphere surrounding the probe are taken on this axis. As a base for angular measure about this axis, the plane containing the target planet, the Sun, and the probe is taken. Note that the probe--planet line is the intersection of this plane with the plane containing the target planet, Earth, and the probe. Figure 2 illustrates the relations outlined above. The projection of the target planet on the unit sphere and consequently the stereographic projection of the target onto the central plane of the unit sphere normal to the probe--target axis will display the angular relations of subtended angle, position of terminator, etc., visible from the probe. The direction vectors from the probe to the sun and earth, regarded as points on the unit sphere, can also be projected onto this equatorial plane. The convention of using solid dots to represent points projected from the hemisphere away from the target planet and open circles to represent points projected from the hemisphere toward the target planet, locates the projected points on the sphere. The exception to this will be of course the target planet projection, which is always oriented by the above selection

of the reference axis.

If the probe is intended to approach the target planet to within a few times its diameter, less than a complete hemisphere of the planet will be visible from the probe. The geometry of this situation is such that the half angle subtended by the planet $\delta = \sin^{-1} a/R$, where a is the target planetary radius, and R is the probe--planet center separation. It can be shown that if the angle between the sun and probe directions at the center of the target is ϕ; given the angle δ, the position of the terminator as seen from the probe will be shown in Figure 3. The Sun--probe--planet plane in this projection contains the sun-planet line; hence the terminator is normal to this plane. If the entire hemisphere of the planet is visible, the intersections of the terminator with the limit of the visible disk lie on a line normal to the sun--probe--planet plane through the reference axis. Close approach causes the intersection to depart from this line in the direction opposite to the sun. The angle $\sigma = \sin^{-1} (\cos \phi \tan \delta)$ can be shown to specify this departure. The angle between the terminator and the probe--target line, measured in the sun--probe--planet plane, is given by $\tau = \tan^{-1} \frac{a \cos \phi}{R - a \sin \phi}$.

Representation of the positions of the sun and earth relative to the probe in the system described above is simply a matter of (1) laying off the angle S of the sun from the reference axis in the sun--probe--planet plane, (2) locating the diameter representing the earth--probe--planet plane with respect to the sun--probe--planet plane by laying out the included angle γ and finally (3) laying out the angle E of the probe--earth direction with respect to the reference axis. In Figure 3, the relations of these angles for one hypothetical probe position are

shown together with the appearance of the target and the various parameters discussed in the preceding paragraph.

Figure 4 illustrates, in a planet-centered stereographic projection, a nominal trajectory for a Venus fly-by supplied by JPL. The positions indicated in the diagram are those for which computed orbital data are available that permit calculation of the parameters necessary for the projection representation discussed above. Relative directions of probe, sun, and earth are shown, the plane of projection being the Sun-Venus-Earth plane. Planet--probe distances could be shown in a probe orbital plane diagram. Such a planet-centered representation might be helpful in interpreting transmitted data from a geographic scan, since transformations of geographic coordinates for scan data positions from one arbitrary system to another can be easily made using stereographic net techniques. For example, one convenient coordinate system for data analysis might be one in which the north geographic pole is taken on the probe--target line with a reference meridian taken in the sun-probe-Venus plane; transformation of positions given in this coordinate system to another may be desirable, say to one in which the equatorial plane is the target orbital plane (Venus) or to one based on the axis of rotation of the target (Mars). Within the limits of accuracy required for analysis of most of the data from a space probe, transformations of this type can be quickly performed, with the additional advantage that the form of representation lends itself to visualization of the angular relations involved.

Figures 5A, B, C, D, E, and F show the results of applying the projection technique to each of the trajectory positions shown in Figure 4.

The path of a hypothetical geographic scan is shown in each of the projections. This scan is oriented at an angle of 15° to the normal to the sun--earth--probe plane. Note the relative motion of the terminator and scan path through the sequence of positions. This trajectory is not coplanar with the sun--Venus--Earth plane, hence the trajectory shown is merely schematic.

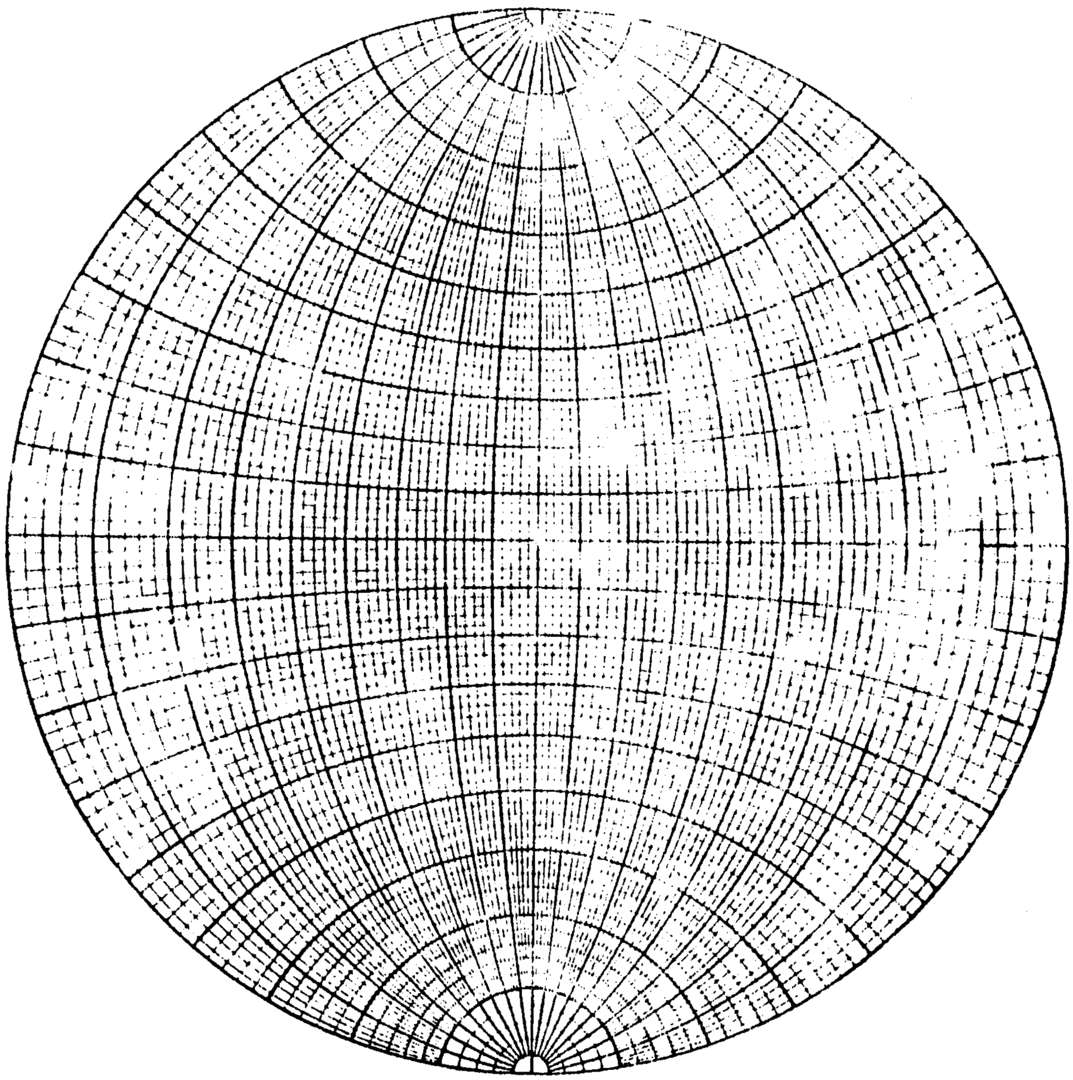


Fig. 1 - The Wulf Stereographic Net

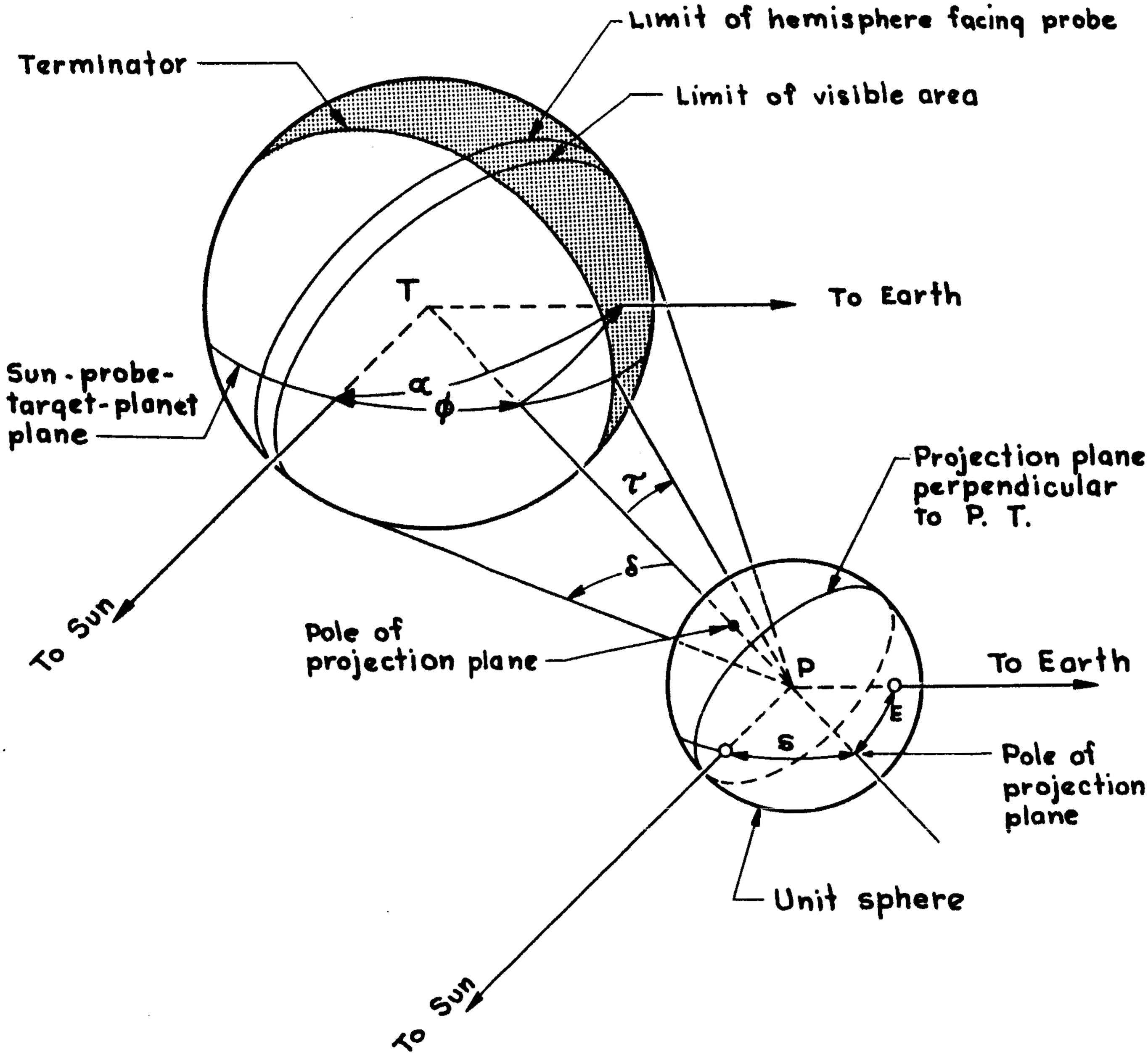


Fig. 2— Probe-centered sphere for stereographic projection of planet and probe environment

○ Open circles = Points projected from unit sphere hemisphere opposite target planet

● Solid circles = Points projected from unit sphere hemisphere facing target planet

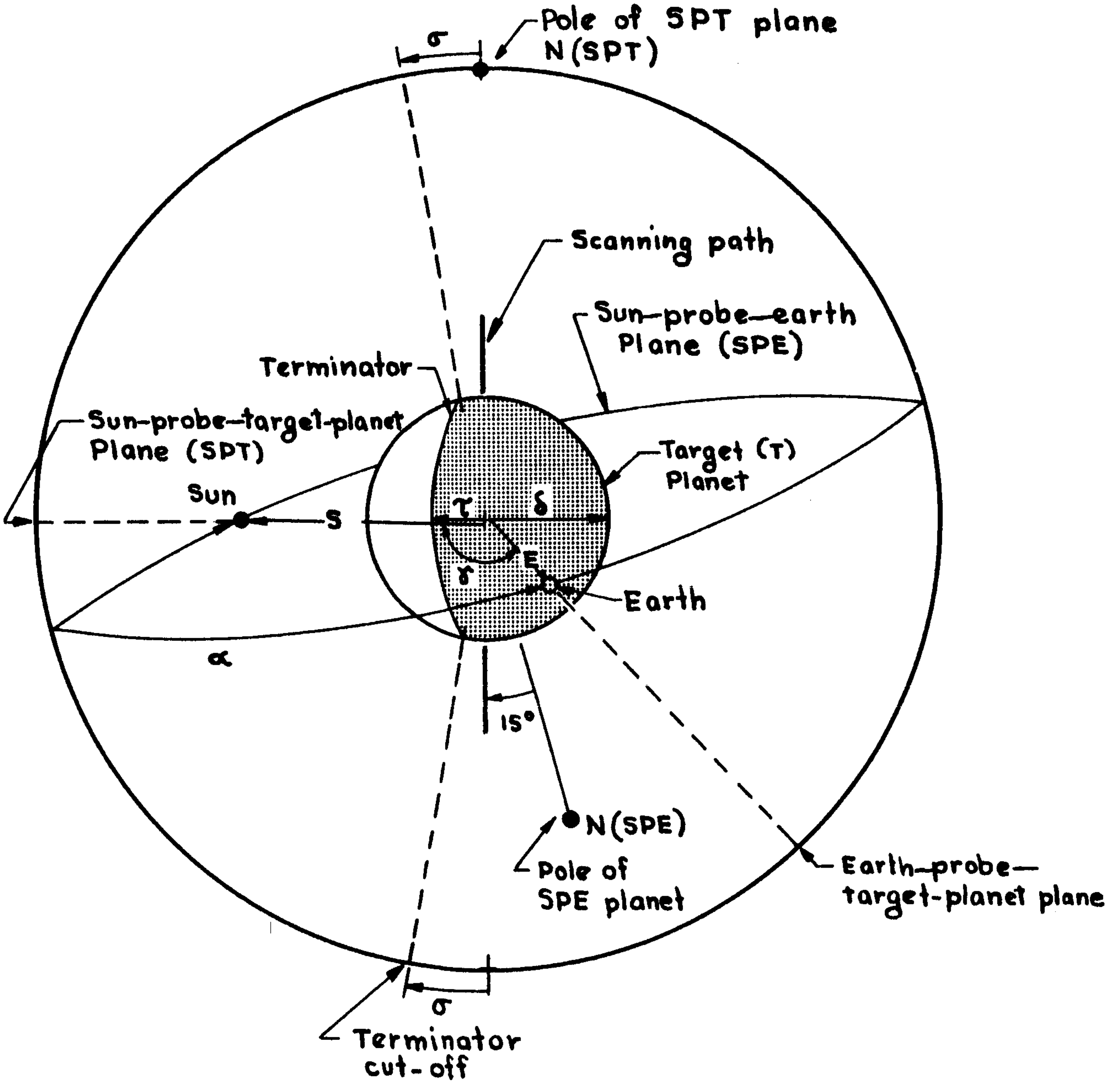
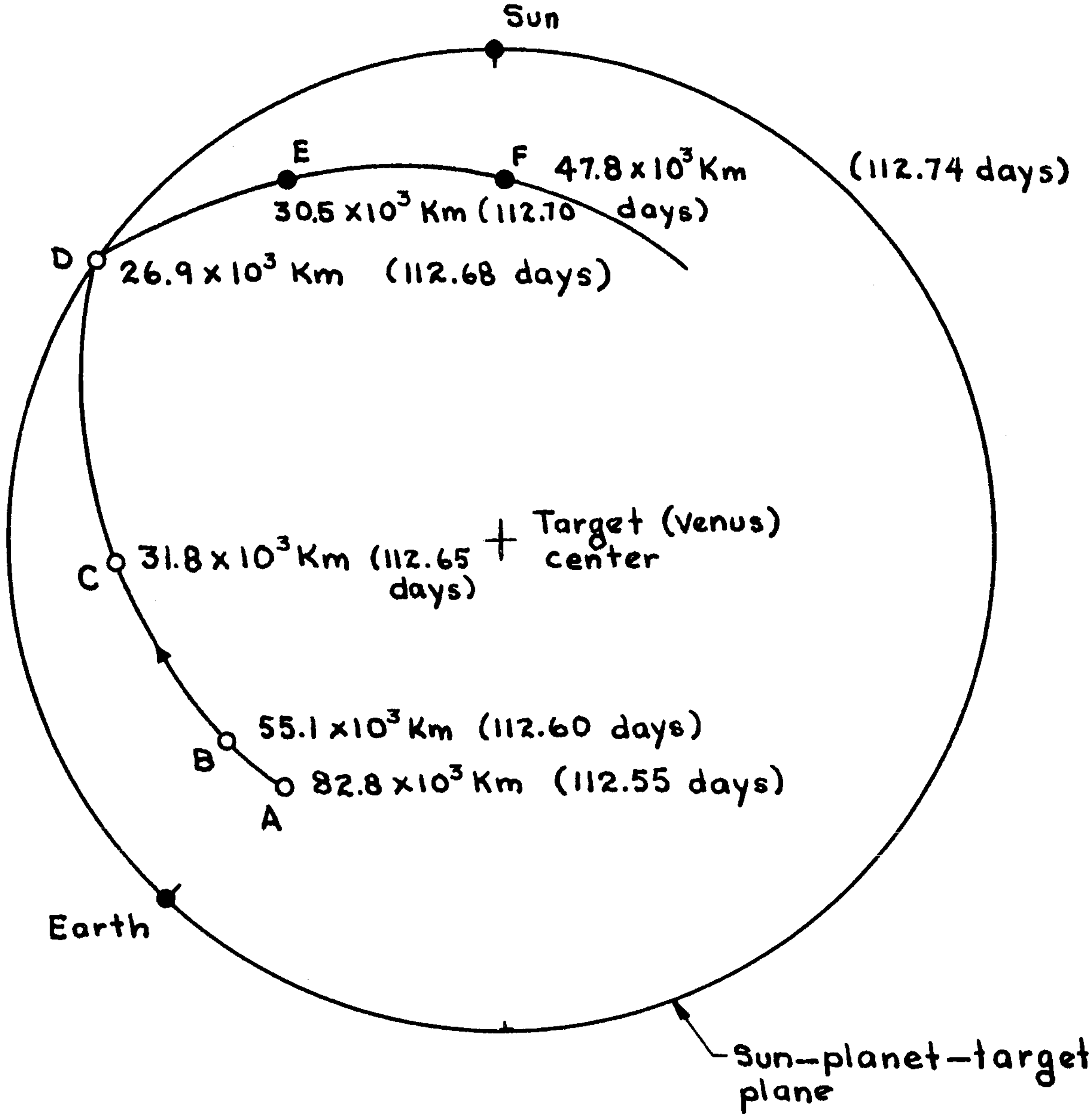


Fig. 3 — Stereographic representation of hypothetical probe environment illustrating parameters presented



Sun-plant-earth phase angle α = 136.431°

Fig. 4 – Target-planet-centered probe-trajectory projection with time and distance of selected points

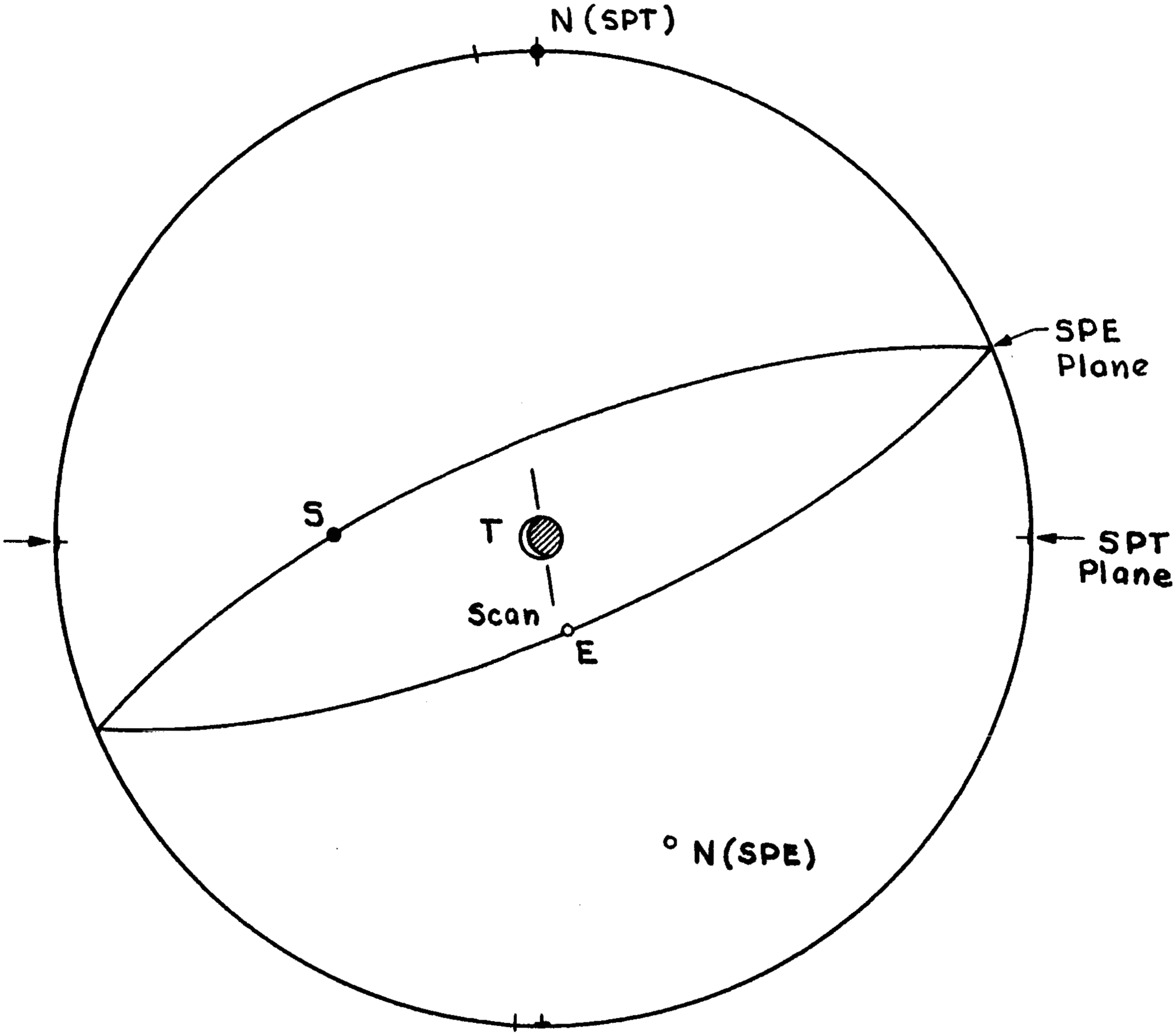


Fig. 5A — For nomenclature see fig. 3
Probe position shown in fig. 4

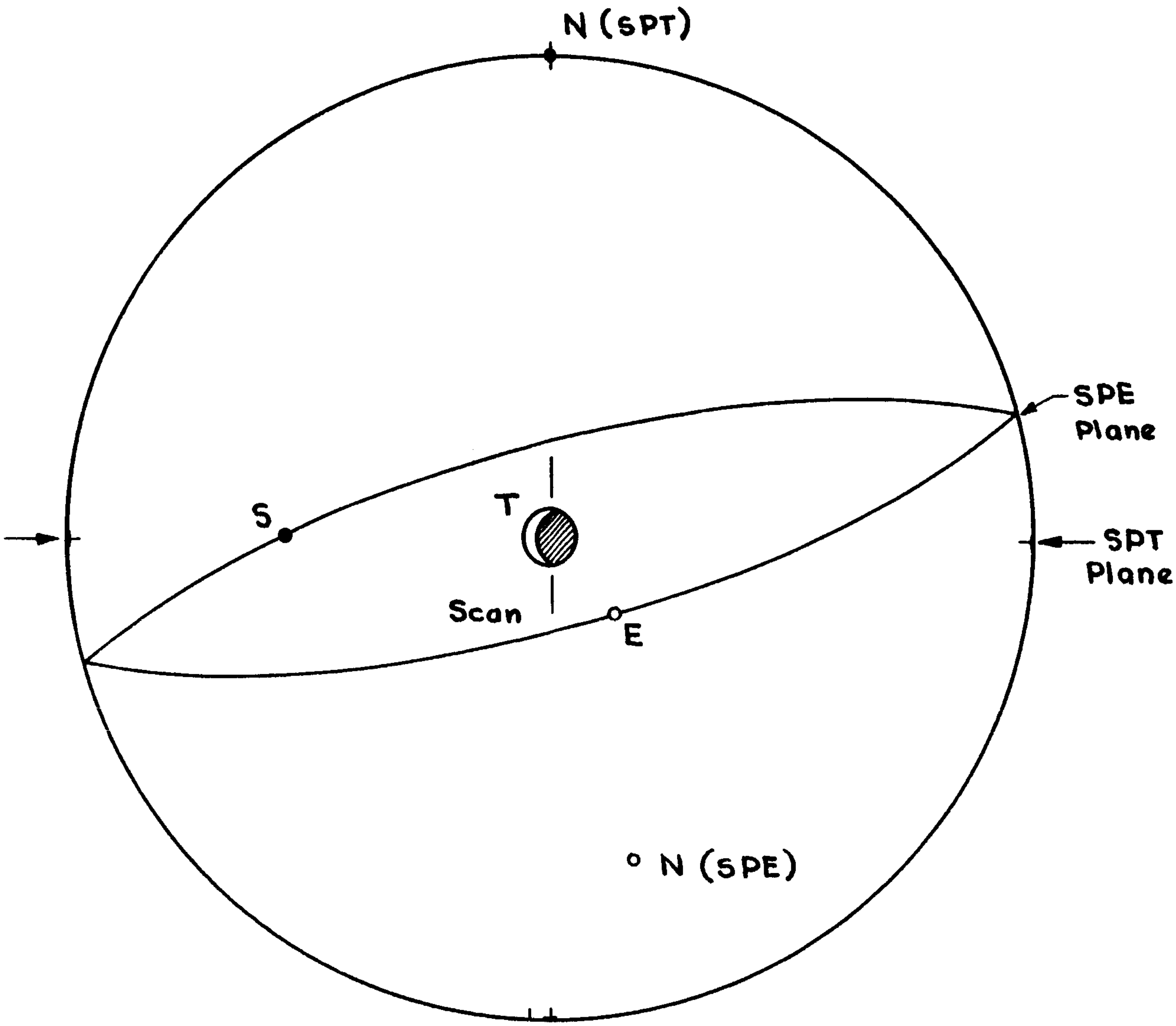


Fig. 5 B

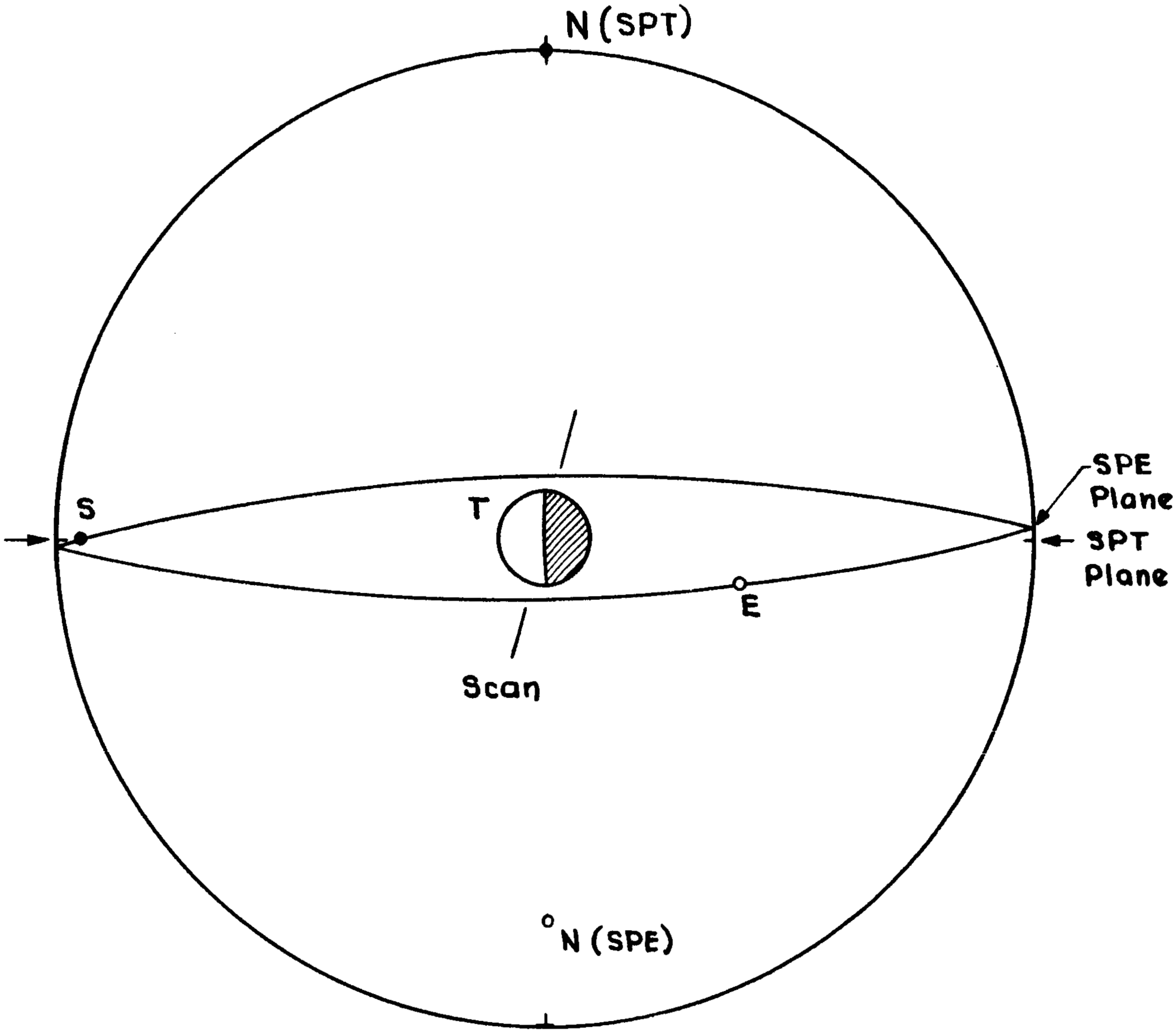


Fig. 5c

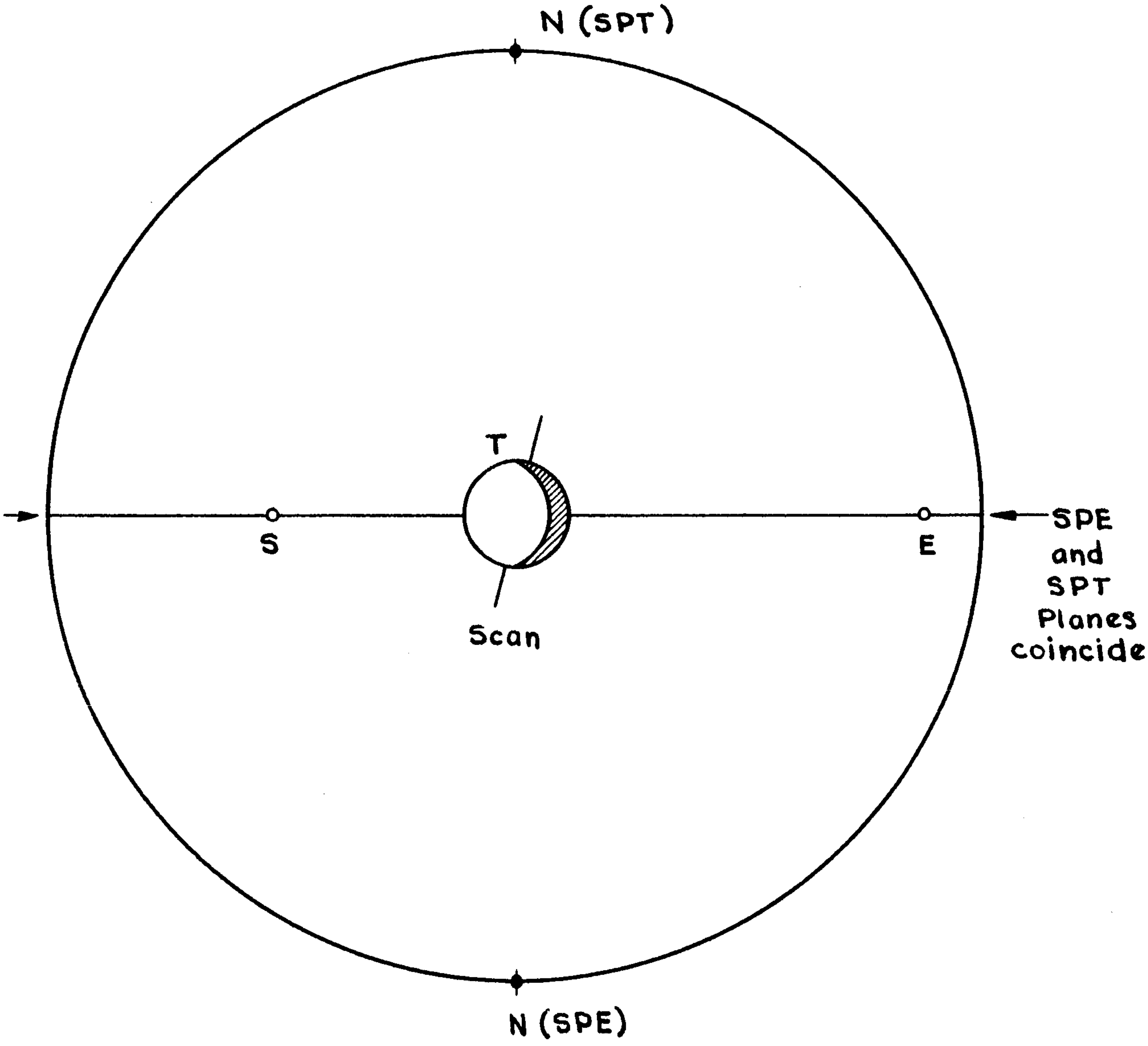


Fig. 5 D

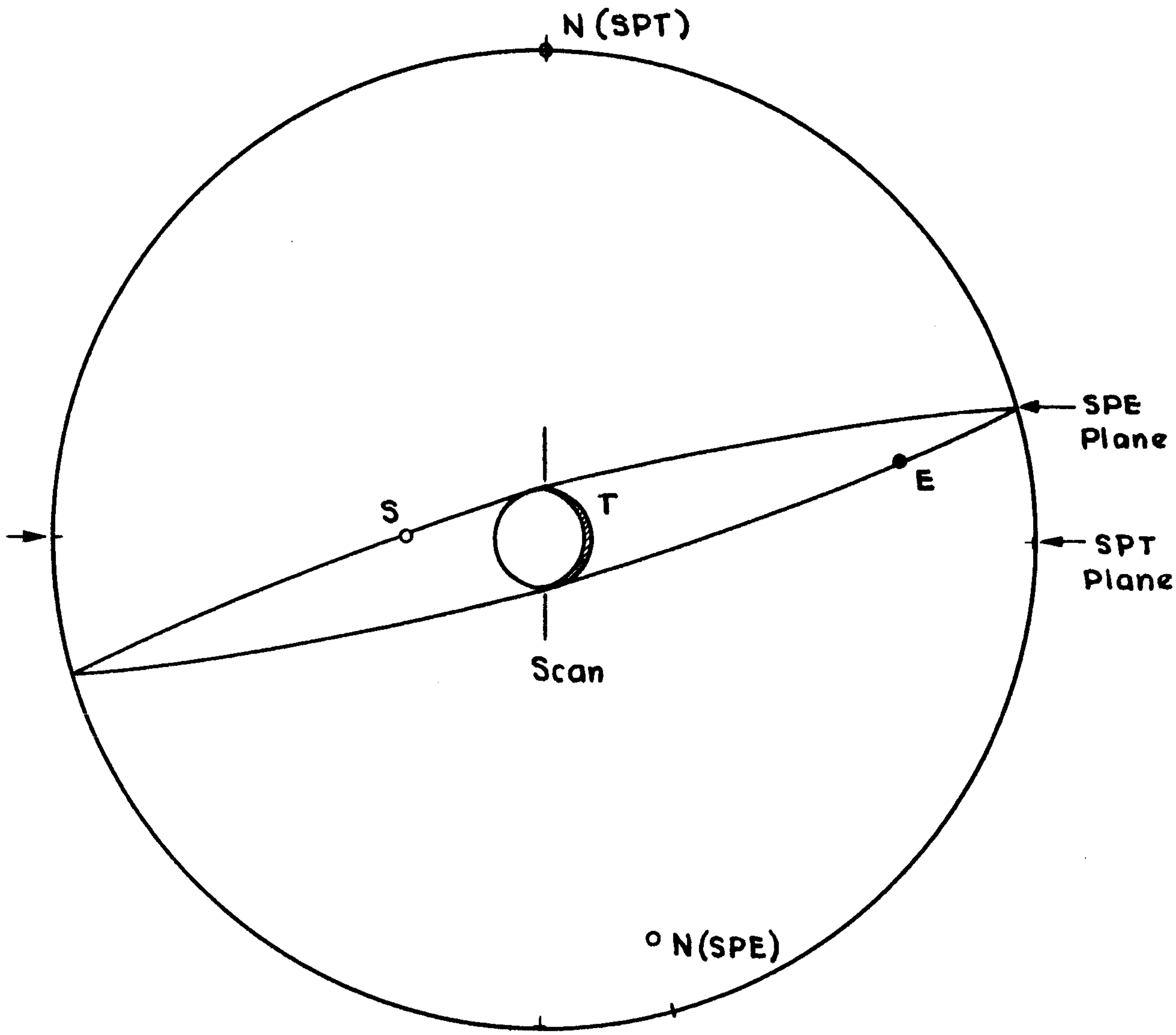


Fig. 5 E

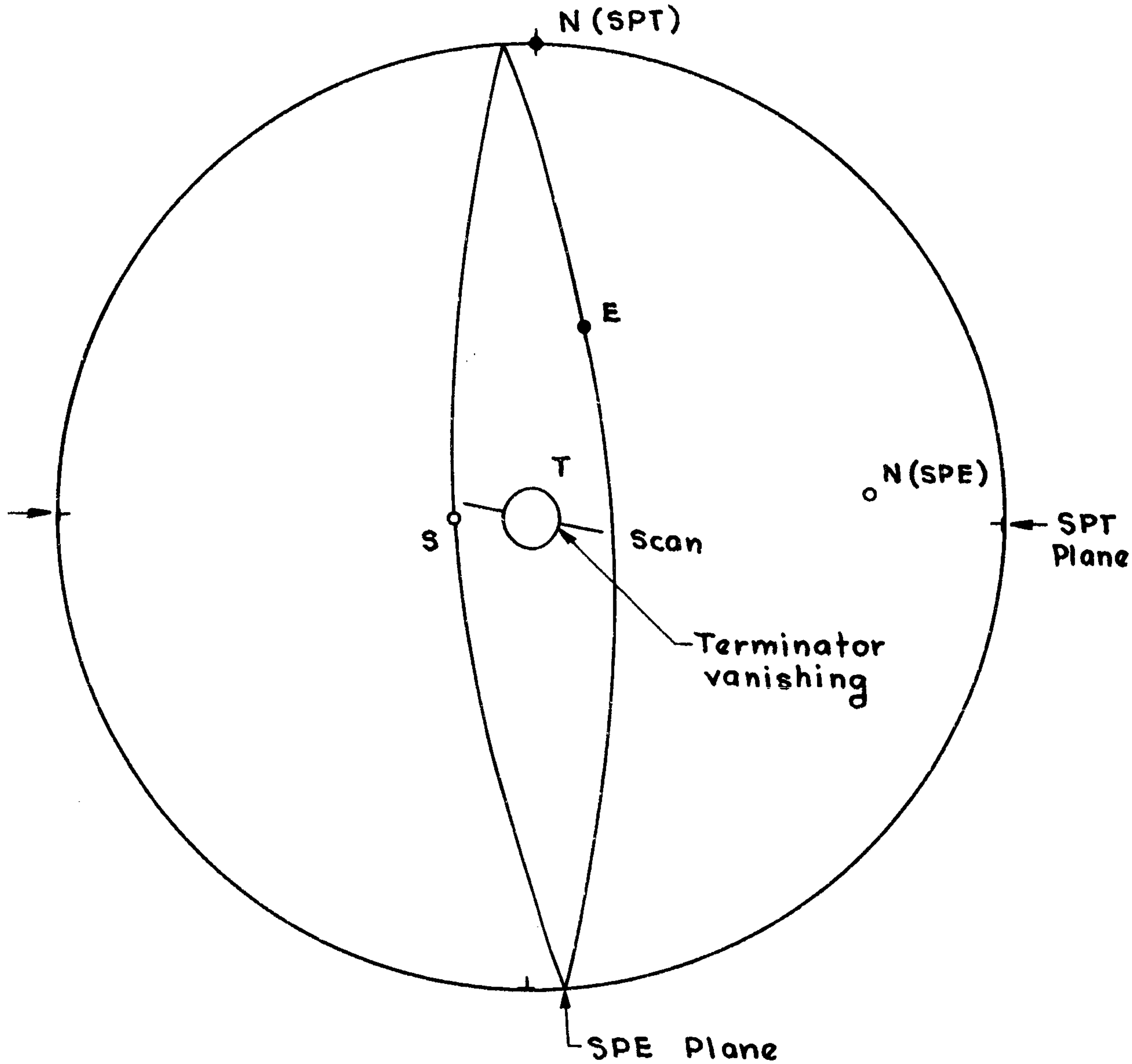

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Fig. 5 F

XI ORBIT DETERMINATION FROM TERMINATOR OBSERVATIONS

T. W. Mullikin

1. Introduction

It will be a very difficult problem to determine the orbit of a probe moving in the vicinity of one of the other planets solely by means of observations of the probe made from the earth. For this reason, methods should be investigated by which the orbit might be derived from observations made from the probe or orbiter itself. One such method is discussed here.

We shall consider a probe in the vicinity of a planet as moving in a two-body orbit. We shall show that it is possible to determine orbit parameters from exact observations of the terminator, the great circle defined by the hemisphere illuminated by the sun, in its various aspects during the motion of the probe in its orbit. Some preliminary considerations of this are given in Sec. IX of RM-2661-JPL.

This method of orbit determination is appealing because of its instrumental simplicity. However, it appears that it is very sensitive to observational errors which, even for perfect instruments, are present in observations of a planet with an atmosphere since then the terminator is not a distinct arc. In the elementary analysis that follows, we do not make a thorough error analysis, nor do we consider how the estimates of orbit parameters might be improved by statistical methods.

As a single method for orbit determination this does not look promising. Observations of the terminator might be useful, however, in providing additional input to a differential-correction procedure for improving estimates of the orbit parameters obtained by other means.

2. The Terminator Angle

We specify a coordinate system (Fig. 1) centered at the planet with the y-z plane in the orbit plane of the planet around the sun. The sun is "at infinity" on the z-axis.*

We neglect the motion in inertial space of this coordinate system as well as solar and planetary perturbations. The orbit of a probe will then be specified by Eulerian angles (Fig. 1) Ω , I and ψ and by the radial distance r from the center of the planet

$$r = \frac{p^2}{1 + \epsilon \cos(\psi - \omega)} . \qquad (2.1)$$

Here p is proportional to the constant total angular momentum per unit mass. If d denotes the distance at closest approach, i.e. r for $\psi = \omega$, then $\epsilon = p^2/d - 1$, which is greater than 1 for a hyperbolic orbit and less than 1 for an elliptic orbit. The orbital motion is related to time by the equation

$$r^2 \frac{d\psi}{dt} = pk . \qquad (2.2)$$

*Nomenclature for this chapter is listed at the end of the chapter.

In the x, y, z coordinate system the terminator of the planet has a parametric representation

$$
\begin{aligned}
x &= R \cos \phi \;, \\
y &= R \sin \phi \;, \qquad\qquad (2.3) \\
z &= 0 \;,
\end{aligned}
$$

where R is the radius of the planet (plus a cloud layer perhaps) (Fig. 2).

The radius vector $\underline{P}$ of the probe will be expressed by

$$\underline{P} = r\,(\mu_1, \mu_2, \mu_3) \;, \qquad (2.4)$$

where the direction cosines μ_1, μ_2, and μ_3 are expressed in Eulerian angles by

$$
\begin{aligned}
\mu_1 &= \cos\psi \cos\Omega - \sin\Omega \sin\psi \cos I \;, \\
\mu_2 &= \cos\psi \sin\Omega + \cos\Omega \sin\psi \cos I \;, \qquad (2.5) \\
\mu_3 &= \sin\psi \sin I \;.
\end{aligned}
$$

We want an expression for the angle γ made at the probe by the vector $\underline{P}$ to the center of the planet and the vector $\underline{Q}$ to a point on the terminator. By vector analysis

$$\tan\gamma = -\frac{|\underline{P} \times \underline{Q}|}{\underline{P} \cdot \underline{Q}} \qquad (2.6)$$

Since

$$\underline{Q} = (R\cos\phi,\ R\sin\phi,\ 0) - \underline{P} \;, \qquad (2.7)$$

this becomes

$$\tan\gamma = \frac{R\left[\mu_3^{\,2} + (\mu_1 \sin\phi - \mu_2 \cos\phi)^2\right]^{\frac{1}{2}}}{r - R\,(\mu_1 \cos\phi + \mu_2 \sin\phi)} \qquad (2.8)$$

This specifies γ as a function of ψ as the probe moves in the orbit given by (2.1) and (2.5), where Ω, I, ω, p and ϵ are parameters and ϕ is a free variable.

Both for practicality of the observation we have in mind and for mathematical simplicity, we will choose ϕ as a function of ψ in such a way that $\underline{Q}$ is the vector to the nearest point on the terminator. Since $\underline{Q}$ is to be a minimum with ϕ, we have

$$\underline{Q} \cdot \frac{d\underline{Q}}{d\phi} = 0 \tag{2.9}$$

or

$$\tan \phi = \frac{\mu_2}{\mu_1} \tag{2.10}$$

This choice of ϕ also puts $\underline{Q}$, $\underline{P}$ and the z-axis in the same plane.

We shall always take γ so that $0 \leq \gamma \leq \pi/2$. Then (2.10) in (2.7) gives

$$\tan \gamma = \frac{R|\beta|}{r - R\sqrt{1 - \beta^2}} \quad , \quad \beta = \sin \psi \sin I. \tag{2.11}$$

We have changed notation from μ_3 to β. We also assume that $I > 0$ so that the orbit does not lie in the terminator plane.

Since the terminator is not visible when the probe enters the cylinder $x^2 + y^2 < R^2$, equation (2.11) is physically meaningful only for $r\sqrt{1 - \beta^2} > R$. We speak of this cylinder as the "shadow".

For an elliptic orbit the angle γ is a periodic function of ψ.

For a hyperbolic orbit the angle ψ ranges through the interval

$$\omega - \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\epsilon}\right) < \psi < \omega + \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\epsilon}\right) \tag{2.12}$$

If we assume that the point of closest approach is on the sunlit side

of the planet so that $0 < \omega < \pi$, then the angle γ has different behavior depending on whether both, one or none of the following inequalities holds:

$$\begin{aligned} \omega &< \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{\epsilon}\right) \\ \omega &> \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\epsilon}\right) \end{aligned} \tag{2.13}$$

In Fig. 3 the different possibilities are illustrated by graphs showing qualitative behavior of $\tan \gamma$ as a function of ψ .

3. Orbit Determination

We assume that the probe has instruments that measure two quantities. (1) One instrument measures the angle θ from the sun to the probe to the center of the planet and therefore the quantity $\beta = -\cos\theta$. (2) Another instrument is offset an angle α from the radius vector of the probe in the sun-probe-planet plane and is oriented always toward the terminator plane. It detects a crossing of the terminator, and records the value of β at which $\gamma = \alpha$. There could, of course, be several of these instruments or one with a variable angle α .

The angle α has to be chosen small enough that four observations are obtained near the terminator plane and outside the shadow.

For the hyperbolic orbit, (i) of Fig. 3 is the only favorable situation for obtaining four good observations for a single offset angle α . In (ii) four observations are possible; however, two of these are for large radial distances where the orbit, because of solar perturbations, is not strictly hyperbolic. With two offset angles, four observations can always be obtained.

Fig. 1

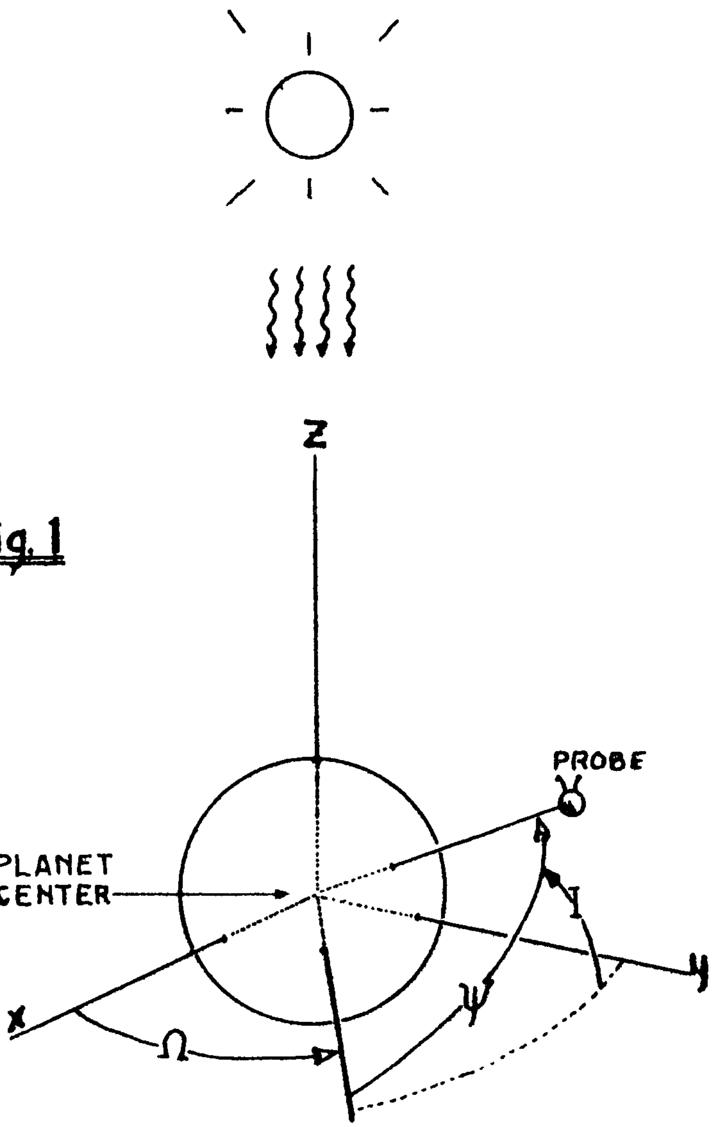


Fig. 2

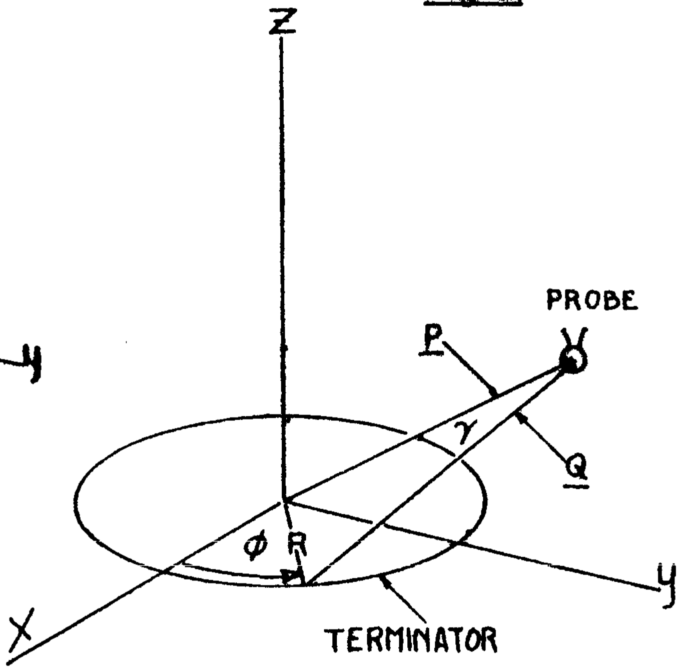
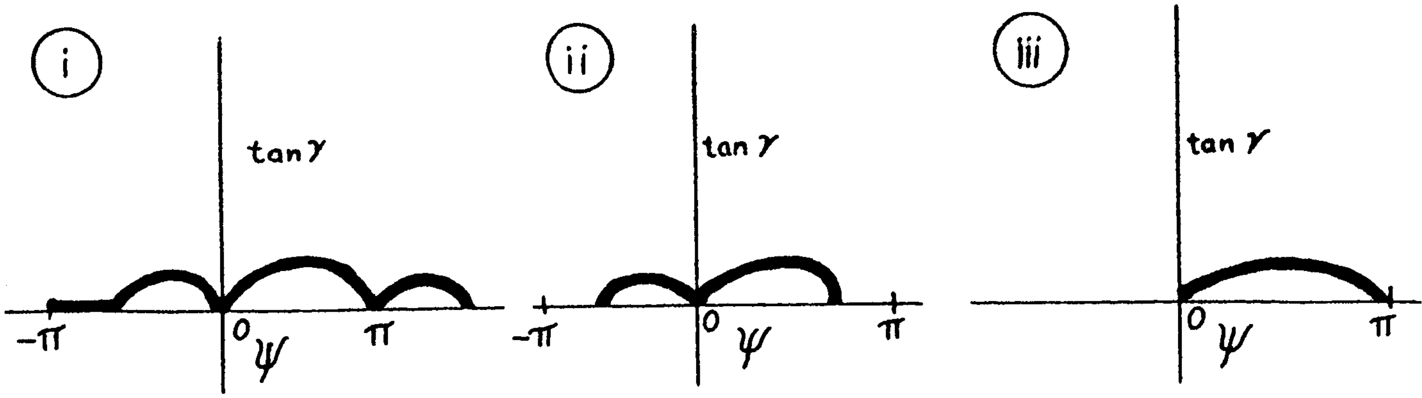


Fig. 3



For elliptic orbits, there is the opportunity of adjusting α, as well as obtaining many observations because in this case the motion is periodic. In fact, repeated calculations of the orbit elements and a determination of their variation could conceivably be used to estimate perturbations, provided the observational errors can be made small.

We suppose then that we have four independent exact observations β_i for which $\gamma = \alpha_i$, $i = 1, \ldots, 4$; the α_i may or may not be distinct. If we assume that R is known in (2.11), we obtain immediately the radial distances

$$r_i = R\left(\frac{|\beta_i|}{\tan\alpha_i} + \sqrt{1-\beta_i^2}\right) . \tag{3.1}$$

From the equations (2.1) and (2.5) we obtain

$$\frac{\epsilon\cos\omega}{\sin I}\left(\sin I\cos\psi_i + \beta_i\tan\omega\right) = \frac{p^2}{r_i} - 1 , \tag{3.2}$$

$$\sin I\sin\psi_i = \beta_i .$$

This is a set of eight equations for the eight unknown ψ_i, $i = 1, \ldots, 4$, ϵ, p^2, ω and I ($0 < I \leq \pi$) . The orbit parameter ω cannot be determined by these observations since the terminator is assumed invariant under rotation about the z-axis.

For brevity we set

$$x_i = \sin I\cos\psi_i , \tag{3.3}$$

and reduce (3.2) to

$$x_i = \pm\sqrt{\sin^2 I - \beta_i^2} , \tag{3.4}$$

$$\frac{\epsilon \cos \omega}{\sin I} = \frac{r_1 - r_2}{r_1 (x_1 + \beta_1 \tan \omega) - r_2 (x_2 + \beta_2 \tan \omega)} , \tag{3.5}$$

$$p^2 = \frac{r_2 r_1 (x_1 - x_2 + (\beta_1 - \beta_2) \tan \omega)}{r_1 (x_1 + \beta_1 \tan \omega) - r_2 (x_2 + \beta_2 \tan \omega)} , \tag{3.6}$$

$$\tan \omega = \frac{r_3 x_3 (r_1 - r_2) + r_1 x_1 (r_3 - r_2) + r_2 x_2 (r_1 - r_3)}{\beta_1 r_1 (r_2 - r_3) + \beta_2 r_2 (r_3 - r_1) + r_3 \beta_3 (r_2 - r_1)} , \tag{3.7}$$

and

$$\begin{aligned} & r_1 x_1 \left[r_2 \beta_2 (r_4 - r_3) + r_3 \beta_3 (r_4 - r_2) + r_4 \beta_4 (r_2 - r_3) \right] + \\ & r_2 x_2 \left[r_1 \beta_1 (r_3 - r_4) + r_3 \beta_3 (r_1 - r_4) + r_4 \beta_4 (r_3 - r_1) \right] + \\ & r_3 x_3 \left[r_1 \beta_1 (r_2 - r_4) + r_2 \beta_2 (r_4 - r_1) + r_4 \beta_4 (r_2 - r_1) \right] + \\ & r_4 x_4 \left[r_1 \beta_1 (r_3 - r_2) + r_2 \beta_2 (r_1 - r_3) + r_3 \beta_3 (r_1 - r_2) \right] = 0 . \end{aligned} \tag{3.8}$$

By repeated squaring of the equation (3.8) and use of (3.4), we obtain a fourth degree polynomial in $\sin^2 I$, which one hopes will have a single real positive root. The correct choice of sign for the x_i is given by the sign of $\cos \psi_i$, which should be obvious from the observations. When the quantity sin I is computed from (3.8) the x_i are determined and then also $\tan\omega$, p^2 and ϵ from the above equations. The computation has therefore been reduced to the solution of a fourth degree polynomial.

4. Error Analysis

We shall not give a thorough error analysis of the computations outlined in the previous section. Rather we indicate the sensitivity of these computations to observational errors and to orbit position relative to the terminator.

First of all it should be observed that, for special orbits, (3.7) does not determine sin I if observations are made with a fixed offset angle α . If $\omega = \frac{\pi}{2}$, then it is easy to show that (3.7) is an identity, since the four observations are really only two because of symmetry of the orbit relative to the terminator plane. This can be avoided, of course, by using different values of the offset angle α , but it indicates the possible singular nature of the equations.

If we assume R known in (3.1), it follows that errors in r are proportional to errors in observations of the angle θ . But the constant of proportionality is essentially $1/\tan \alpha$, which is large since the observations are made near the terminator plane where α is near zero. The relative error $\frac{|\Delta r|}{r}$ is not small either, since $\beta = -\cos \theta$ and θ is near $\pi/2$. Similar statements hold for errors in p^2 .

NOMENCLATURE

For Chapter XI

d = radial distance at closest approach to planet center

I = inclination angle (Eulerian angle)

k = $\sqrt{\text{universal constant of gravitation times mass of planet}}$

$\underline{P}$ = vector from probe to planet center

$\underline{Q}$ = vector from probe to point on terminator

p = $\frac{1}{k}\cdot$ (angular momentum per unit mass)

R = radius of planetary disk

r = radial distance of probe to planet center

x_1 = $\sin I \cos \psi_1$

α = offset angle of terminator detector

β = same as μ_3

γ = angle between $\underline{P}$ and $\underline{Q}$

ϵ = orbital eccentricity

θ = sun—probe—planet angle

μ_1, μ_2, μ_3 = direction cosines of $\underline{P}$

ϕ See Fig. 2.

ψ See Fig. 1. (Eulerian angle)

Ω = node (Eulerian angle)

ω = argument of perifocus